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# Spatial Extension of Quantum Gravitational Particles in $R^4 \times K^N$

**Gerald Rosen**[1]

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It is observed that the magnitude relation $ma^2 \approx \hbar l_P/c$ holds if the non-Euclidean incremental spatial volume associated with a fundamental particle of mass $m$ and radius $a$ is characteristically quantum gravitational in a Kaluza-Klein or superstring $R^4 \times K^N$. Here $R^4$ is the four-dimensional Riemannian space-time of general relativity and $K^N$ is a small-scale, compact, $N$-dimensional space of characteristic quantum gravitational volume $l_P^N$, with $l_P \equiv (\hbar G/c^3)^{1/2} = 1.61 \times 10^{-33}$ cm denoting the Planck length. For the electron and electron neutrino (assumed to possess nonzero mass bounded empirically by $m_{\nu_e} \lesssim <30$ eV) the derived magnitude relation $a \approx (\hbar l_P/mc)^{1/2}$ yields the estimates $a_e \approx 2.5 \times 10^{-22}$ cm and $a_{\nu_e} \gtrsim 3.3 \times 10^{-20}$ cm, spatial extensions which may be detectable by way of fine-scale effects in SSC experiments.

## 1. INTRODUCTION

Recently it has been shown (Rosen, 1987) that the Einstein field equations

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

are equivalent to the physicogeometric statement: A local spherical region of radius $r$ containing energy with moment of inertia $I$ has the physical volume

$$V(r) = (4\pi/3)(r^3 + GI) \tag{2}$$

in an associated free-falling (nonrotating, timelike geodesic) frame of reference. Here Newton's constant is $G = 7.41 \times 10^{-29}$ cm/g in a system of physical units such that the speed of light in vacuum equals unity ($c = 1$). In particular, for a uniform spherical distribution of radius $a$ and total energy

[1]Department of Physics, Drexel University, Philadelphia, Pennsylvania 19104.



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$m$, one has $I = \frac{2}{5}ma^2$; therefore, the non-Euclidean incremental spatial volume $\Delta V \equiv V(a) - 4\pi a^3/3$, i.e.,

$$\Delta V = (8\pi/15)Gma^2 \tag{3}$$

is attributed to a mass $m$ of radius $a$, according to general relativity and formula (2). Conversely, if (2) or (3) is valid for any spherical region of arbitrary radius containing a uniform distribution of energy in a free-falling frame of reference, then the gravitational field equations (1) must hold through the space-time region. The non-Euclidean character of physical space, as prescribed by general relativity, is expressed in an equivalent quantitative fashion by (3).

## 2. QUANTUM GRAVITATIONAL PARTICLES IN $R^4 \times K^N$

In contemporary studies of the Kaluza–Klein and superstring models (Lovelace, 1984; Fradkin and Tseytlin, 1985; Ne'eman and Sijacki, 1986), space-time is viewed as a $(4+N)$-dimensional manifold with the structure $R^4 \times K^N$, the direct product of the four-dimensional Riemannian space-time of general relativity and a small-scale, compact, $N$-dimensional space of characteristic quantum gravitational volume $l_P^N$, where $l_P \equiv (\hbar G)^{1/2} = 1.61 \times 10^{-33}$ cm is the Planck length. The physical distinction between the three spatial dimensions of $R^4$ and the $N$ spatial dimensions of $K^N$ is likely to become indefinite on a scale of order $l_P$, due to quantum gravitational fluctuations or a possible discrete aspect to space-time that underlies spatial dimensionality [see, for example, Bombelli *et al.* (1987) and work cited therein]. If all $N+3$ spatial dimensions enter on the same footing at the smallest physical scales, then the small-scale equivalence of the $N+3$ spatial dimensions suggests that the non-Euclidean incremental $(N+3)$-dimensional spatial volume attributed to a fundamental particle of mass $m$ and radius $a$ is also characteristically quantum gravitational in (a $t = \text{const}$ hypersurface of) $R^4 \times K^N$. If this quantum gravitational feature is indeed manifest at the smallest spatial scales, it follows that the $(N+3)$-dimensional volume must be of order $l_P^{N+3}$ and the magnitude relation $\Delta V \sim l_P^3$ must hold for the quantity (3) in $R^4$. Thus, such a characteristically quantum gravitational particle in $R^4 \times K^N$ requires the magnitude relation $Gma^2 \approx l_P^3$, or equivalently

$$ma^2 \sim \hbar l_P \tag{4}$$

to hold as a quantum constraint on the product of the particle mass times its radius squared.

For the electron and electron neutrino (assumed to possess nonzero mass bounded empirically by $m_{\nu_e} \lesssim 30$ eV) the spatial extension estimate that follows from (4),

$$a \approx (\hbar l_{\rm P}/m)^{1/2} \tag{5}$$

yields

$$a_e \sim 2.5 \times 10^{-22}\ \text{cm} \tag{6}$$

$$a_{\nu_e} \lesssim 3.3 \times 10^{-20}\ \text{cm} \tag{7}$$

The latter very small spatial extensions may be detectable by way of finite-scale effects in SSC experiments, with the lengths on the right side of (6) and (7) giving the scales at which measurable departures from the predictions of standard (point-charge) QED and electroweak theory may be expected to show up.

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